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Electron paramagnetic resonance of phosphate-derived radicals in KCI

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Received 12 March 1992, in final form 7 May 1992

Abstract. We examined the EPR spectrum of KCI single crystals doped with either K_3PO_4 or P_2O_5 after x-ray or neutron irradiation. Two new related O⁻ defects and two phosphorus-containing radicals were detected. One of the latter is identified as a PO_2^{2-} radical in an anion vacancy.

1. Introduction

During our investigations of isoelectronic paramagnetic oxides and oxyions of the non-metals (e.g. NO; CO₂⁻, NO₂; CO₃⁻, NO₃; etc) in alkali halides we found it striking that very little is known about phosphate-derived radicals (PO, PO₂, PO₃, PO²⁻, etc). Indeed ³¹P (nuclear spin $I = \frac{1}{2}$) has a natural abundance of 100% which allows rather easy identification and inexpensive possibilities for studying the ³¹P hyperfine interactions. On the other hand a nearly overwhelming number of nitrate-and nitrite-derived paramagnetic defects were reported in alkali halides [1–4].

Our interest in the phosphorus-centred radicals originates also from our studies of biological and synthetic carbonate-containing hydroxyapatites $Ca_{10}(OH)_2(PO_4)_6$. Even in these lattices, very few phosphate-derived paramagnetic defects contribute to the EPR spectra except for PO_4^{2-} after irradiation at low temperatures (e.g. 77 K). A general literature search, however, yields EPR data concerning PO_4^{2-} , PO_3^{2-} , PO_2^{2-} , PO_3 and HPO_2^{-} [5–10].

In this paper we discuss our results on both Kyropoulos- and Bridgman-grown KCl crystals doped with either K_3PO_4 or P_2O_5 . A comparison with other isoelectronic defects is made.

2. Experimental details

On the one hand, KCl crystals were grown using the Kyropoulos method. 1.6 wt% $K_3PO_4.3H_2O$ (Merck) was dried at 200 °C and added to the alkali halide powder. On the other hand, KCl single crystals were prepared by the Bridgman method. Doping with either 0.1 wt% $K_3PO_4.3H_2O$ or P_2O_5 was carried out after drying the starting materials at 250 °C.

The x-irradiations were performed with a Philips tungsten anticathode x-ray tube, operated at 60 kV and 40 mA. Typical irradiation doses are 10-50 kGy. Neutron

irradiations were carried out at room temperature at a mean energy of 0.025 eV for about 1 h in a flux of 1.6×10^{12} cm⁻² s⁻¹.

The EPR spectra were recorded using a Bruker ESP300 X-band spectrometer with 200 mW maximum power available. The magnetic field was modulated at 100 kHz with a peak-to-peak amplitude of 0.1 mT. All spectra were normalized to the same frequency, namely 9.47 GHz. Temperatures down to approximately 2 K could be realized by means of an Oxford Instruments ESR10 flow cryostat.

3. EPR results

3.1. Kyropoulos crystal KCl:K₃PO₄.3H₂O (1.6 wt%)

In spite of intensive searching, no EPR spectra due to phosphorus-centred radicals could be detected after x-irradiation. The latter was performed at room and at liquid-nitrogen temperature. Since, for example in CaCO₃ [5], PO_3^{2-} was produced after neutron irradiation, we tried such a treatment also. Again no phosphate-derived radicals were found. Nevertheless a spectrum (labelled S1) exhibiting no hyperfine structure could be studied at about 2 K and 0.01 mW microwave power. In figure 1 the angular dependence of the S1 resonances is presented with **B** in a {100} plane, showing monoclinic **g**-tensor symmetry; one **g** tensor axis (y axis) is along $\langle 001 \rangle$ while the x and z axes are tilted 5° away from $\langle 110 \rangle$ and $\langle 1\overline{10} \rangle$, respectively (table 1).



Table 1. g-tensor of defect S1. The direction cosines L, M and N are referred to the crystal axes.

	g	L	М	Ν
C	2.5470	0.642 788	0.766 045	0
2	2.2539	0	0	1
z	1.9056	0.766 045	-0.642 788	0



Figure 2. Angular variation in the S2 spectrum in a $\{100\}$ plane. The full curves are calculated with the **g**-tensor in table 2.

3.2. Bridgman crystal KCl:K₃PO₄.3H₂O (0.3 wt%)

In these crystals, several spectra could be observed after x-irradiation at room temperature (XRT). One spectrum (labelled S2) is best detectable at 5 K and 1 mW microwave power and exhibits orthorhombic g-tensor symmetry. The g-tensor principal axes are oriented along $\langle 110 \rangle$, $\langle 1\overline{10} \rangle$ and $\langle 001 \rangle$ (table 2 and figure 2).

Table 2. g-tensor of defect S2.

$g_x(110)$	$g_y \langle 001 \rangle$	$g_z \langle 1 \overline{1} 0 \rangle$	
2.5453	2.2855	1.9071	

The close resemblance between the **g**-tensor principal values of S1 and S2 is striking. Only the orientations of the g_x and g_z axes are slightly different for both centres.

In the same crystal, two other spectra are characterized by a hyperfine splitting with $I = \frac{1}{2}$, probably originating from phosphorus-centred radicals with one ³¹P nucleus. One of these spectra (P1) is weakly visible at room temperature but the variation in the line positions is small compared with the linewidth so that the spin-Hamiltonian parameters could only be roughly estimated:

$g_{\perp} = 2.003 \pm 0.001$	$A_{\perp} = 170 \pm 30 \mathrm{MHz}$
$g_{ } = 2.002 \pm 0.001$	$A_{\parallel} = 112 \pm 30 \mathrm{MHz}.$

The P1 spectrum depends in a very peculiar way on the irradiation time. It is not visible without XRT but after irradiations longer than about 2 min (approximate dose, 2 kGy) the intensity of the spectrum decreases again.

The second spectrum with hyperfine structure (P2) was best detectable near 20 K and 0.01 mW. We averaged the EPR signals five times to increase the signal-to-noise ratio. In figures 3(a) and 3(b) the spectra are presented for $B||\langle 100 \rangle$ and $B||\langle 110 \rangle$, respectively. In figure 4 the experimental angular variation is fitted with the spin-Hamiltonian parameters of table 3.

The choice of the signs for the hyperfine (HF) parameters will be discussed below. The same P2 spectrum was found in Bridgman crystals doped with P_2O_5 .



Figure 3. (a) EPR spectrum of defect P2 with $B||\langle 100 \rangle$. (b) EPR spectrum of defect P2 with $B||\langle 110 \rangle$. The central line around g = 2.0023 does not belong to the P2 spectrum.



Table 3. Spin-Hamiltonian parameters of defect P2.

		A	
	g	(MHz)	
x(110)	2.0021	458.8	
y(110)	2.0071	98.3	
z(001)	2.0050	117.9	
Average	2.0047	225.0	

4. Discussion

Although both crystal growth methods are rather different (air contact for Kyropoulos, and vacuum for Bridgman), it remains unclear why the S1 spectrum occurs only in

Kyropoulos-grown crystals while the opposite is true for the S2, P1 and P2 defects. As the P1 and P2 spectra are weak, it might be only a matter of concentration in this case. As will become clear, S1 and S2 are in fact the same defect although we do not understand the reason for the slightly different **g**-tensor axes. The above certainly illustrates the fact that the production of paramagnetic phosphate centres in alkali halides is in general poorly understood.

4.1. SI and S2 centres

As the spectra of these two centres show no doublet splitting due to the ³¹P $(I = \frac{1}{2})$ nucleus, oxygen centres are most probable in view of the doping with K_3PO_4 . In addition we suspect that the same defect accounts for both the S1 and the S2 spectra because of their very similar **g**-tensors. A glance at the literature reveals that O_2^- and O_3^- ions whose **g**-tensors have also D_{2h} symmetry can be ruled out; the **g**-tensor of O_2^- is nearly axial with two principal g-values smaller than $g_e = 2.0023$ [11], while the **g**-tensor of O_3^- is characterized by a small anisotropy and three principal values larger than g_e [12, 13]. On the other hand the **g**-tensor values of S1 and S2 correspond very well qualitatively and even quantitatively to those of some O^- centres in alkali halides [14, 15].

The theory for a p^5 (²P) configuration was well established by Vannotti and Morton [16] and yields the following principal **g**-tensor values:

$$g_x = g_e(-A^2 + B^2 + C^2) + 4lBC$$

$$g_y = g_e(A^2 - B^2 + C^2) + 4lAC$$

$$g_z = g_e(-A^2 - B^2 + C^2) + 4AB$$

where A, B, C and l are adjustable parameters (for more details see [16]).

As could be expected, both g-tensors can be perfectly interpreted in terms of the above equations. Hence we suggest that both S1 and S2 centres are essentially O⁻ ions in slightly different surroundings. Since the g-tensor principal axes of the S2 centre are simple crystallographic orientations, we propose a substitutional site for this defect, in agreement with the KCI:S⁻(110) model of Vannotti and Morton [16]. In fact it is baffling to see that the g-tensors of the latter S⁻(110) centre and of the S2 model are nearly identical. By doping with sulphur enriched with ³³S ($I = \frac{3}{2}$) the identification of the Vannotti-Morton S⁻(110) model was firmly established. Ascribing S2 to some sulphur centre due to accidential contamination is nevertheless highly improbable because the S2 lines are very strong and no trace of optical S⁻ bands were found. Furthermore a comparable coincidence between the KCI:O⁻₂ [11] and KCI:S⁻₂ g-tensor values and g-tensor axes was already mentioned in [17]. In that case, however, both ¹⁷O [11] and ³³S enrichment [17] could exclude any misidentification.

As far as the S1 resonances are concerned, we think that they arise from a substitutional O^- in perturbed surroundings, caused by radiation damage following neutron irradiation. More detailed models seem too speculative in view of the lack of experimental information such as ENDOR data.

4.2. Introductory remarks concerning the P1 and P2 centres

Both the P1 and the P2 EPR spectra can be interpreted in terms of radicals containing only one ³¹P nucleus (natural abundance, 100%). A priori eight reasonable possibilities can be considered: PO, PO₂, PO₃, PO₄ and PO²⁻, PO₂²⁻, PO₃²⁻, PO₄²⁻. Cations

are improbable in view of the doping procedure and in view of the EPR results on related molecules and molecular ions.

The PO_3^{2-} and PO_2 possibilities can be safely eliminated because of their large hyperfine splittings. This can be simply explained as for both radicals the unpaired electron occupies a molecular orbital with A₁ symmetry, yielding large ³¹P HF values due to the contribution of the 3s_p atomic orbital. Experimentally one finds for example CaCO₃:PO₃²⁻ splittings exceeding 1700 MHz [5]. The PO₂ molecule is isoelectronic with for example NO₂ and CO₂⁻ which have already been studied in alkali halides and particularly in KCl [4, 18]. Considering table 4, we expect the HF splittings of phosphorus-containing radicals to be roughly six and three times larger than the corresponding nitrogen- and carbon-containing radicals, respectively. As the splittings of ¹³CO₂⁻ already exceed 300 MHz, a PO₂ model for either P1 or P2 is very improbable too.

Next to the pair $PO_2 - PO_3^{2-}$ we shall consider the $PO_3 - PO_4^{2-}$ and $PO - PO_2^{2-}$ pairs. These pairs differ only in an O^{2-} ion with a noble gas configuration and are often mixed up because they are nearly indistinguishable on the basis of simple theoretical arguments (see later and [19, 21-24]).

Table 4. Calculated hyperfine couplings and spin-orbit coupling constants for a few nuclei with $J \neq 0$ [19, 20], where $A_0 = (\mu_0/4\pi)(8\pi/3)(g_N\beta_Ng_e\beta/h)|\Psi_s(0)|^2$ and $B_0 = (\mu_0/4\pi)\frac{2}{5}(g_N\beta_Ng_e\beta/h)\langle r^{-3}\rangle_p$.

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	A ₀ (MHz)	B ₀ (MHz)	λ (cm ⁻¹)
¹³ C	3108.66	90.56	29
¹⁴ N	1538.22	47.73	76
17 O	-4626.26	- 143.69	151
31 P	10 188.28	286.00	299
³³ S	2728.01	78.97	382
⁷⁵ As	9531.32	254.45	1550
⁷⁷ Se	13 419.78	377.85	1688

 PO^{2-} is isoelectronic with O_2^- , S_2^- , Se_2^- and SSe^- with a **g**-tensor behaviour clearly different from those found for P1 and P2; these four ions have two *g*-values smaller than g_e in alkali halides [11, 25, 26]. In the literature, nothing is known about PO_4 so that we shall confine ourselves to the discussion of the $PO_3-PO_4^{2-}$ and $PO-PO_2^{2-}$ pairs. As will be argued below, the P1 spectrum probably belongs to the former, and the P2 spectrum to the latter pair. Figure 5 is very illustrative in this respect and will be further explained in the section concerning the P2 centre.

4.3. P1 centre

From the literature it is known that the PO₃ and PO₄²⁻ radicals have smaller HF values than the PO-PO₂²⁻ pair to be discussed in the next section. PO₃ is isoelectronic with CO₃⁻ and NO₃ which we know from the literature and our own experience in alkali halide $\sin_1\beta$ crystals. The P1, CO₃⁻ and NO₃ defects are all formed after XRT, are visible at RT and decay rather quickly. The latter two radicals which have 23 valence electrons are also both characterized by small HF splittings and anisotropy $\langle\langle A \rangle \simeq 30$ MHz and 10 MHz for CO₃⁻ and NO₃, respectively) which can be adequately explained by their ²A'₂ ground state. The non-zero isotropic HF splitting arises



Figure 5. s and p fractions of the unpaired spin density for molecules and molecular ions isoelectronic with PO₃, PO_4^{2-} (block 1), PO₂ (block 2), PO_3^{2-} (block 3) and PO, PO_2^{2-} (block 4). In block 4 the P2 values are indicated for the + and - choice of HF constants.

completely from spin polarization effects. As discussed by Atkins and Symons [19], the same is true for AB_4 species with 31 valence electrons such as $PO_4^{2^-}$. Isotropic HF splittings ranging from 55 to 140 MHz were reported [5, 10], in agreement with the P1 HF tensor. As our experimental results are only partial we shall not speculate about either a PO_3 or a $PO_4^{2^-}$ model. Nevertheless we believe that the considerations about PO_3 and $PO_4^{2^-}$ are also useful to exclude them as possibilities for the P2 centre with a definitely larger isotropic HF splitting of about 225 MHz and especially a strong anisotropy of the hyperfine tensor.

4.4. P2 centre

By excluding the other possibilities we are left with PO and PO_2^{2-} as possible candidates for the P2 centre. However, we shall corroborate this assignment by a positive argument also.

The spin Hamiltonian of the P2 radical (table 3) is very similar to that of a nitrogen-containing radical that was found in KCl (and KBr) by several workers [1–3]. The spin Hamiltonian of this AN1 centre (assigned to an interstitial NO molecule) is included in table 5.

Considering table 4 it is found that the A-tensor of the P2 centre is to a very good approximation the same as for AN1, taking into account the proportionality factor between the A_0 - and B_0 -values for ³¹P and ¹⁴N. Also the g-tensor principal values are in qualitative agreement. Only the orientation of minimum g and maximum A corresponds to (110) for our P2 defect at variance with the AN1 defect. Provided that this model for AN1 [1-3] is correct, one is tempted to identify the P2 centre as a PO molecule oriented along (110) in a positive or negative vacancy. Without going into details we mention, however, that the assignment of several spectra to NO (including AN1) has been strongly criticized. Many workers agree that an NO₂²⁻

	$g\langle 110\rangle$	g(110)	g(001)	A(110) (MHz)	A(110) (MHz)	A(001) (MHz)	(A) (MHz)
ANI (KCI)	2.0070	2.0099	2.0038	19.7	14.1	86.6	40.0
O ₃ (KBr)	2,0027	2.0180	2.0113	_	_	_	—
SŐ; (KCI)	2.0025	2.0110	2.0071	147	-20	24	35
SO ⁵ (BaSO ₄)	2.0032	2.0128	2.0104	151	- 20	-23	36
SeŐ; (KCI)	1.9989	2.0367	2.0165	688	-237	-254	65
SeO 🗍 [29]				670	240	-269	54
P2 (KCI)	2.0021	2.0071	2.0050	458	±98	±118	225
AsO_2^{2-} (CaCO ₃)	1.9991	2.0150	1.9910	614	±152	±173	313
PO_2^{2-} (BaSO ₄)	1.9992	2.0037	1.9992	840	±258	±255	451
$PO_2^{\tilde{2}-}$ (P ₂ O ₅)	2.0014	1.9989	1.9989	994	±122	±122	413

Table 5. Comparison of isoelectronic defects ($\langle A \rangle$ is the isotropic part of the HF tensor [1, 7, 13, 28-30]. The $\langle 110 \rangle$, $\langle 1\overline{1}0 \rangle$ and $\langle 001 \rangle$ axes apply only to the centres in alkali halides.

model is preferable (see, e.g., [1-3, 10, 19, 22, 27]).

The NO_2^{2-} ion with 19 valence electrons is isoelectronic with for example PO_2^{2-} , SO_2^{-} and O_3^{-} and the unpaired electron is expected to reside in a $b_1(\pi^*)$ molecular orbital:

$$|b_1\rangle = -c_1 p_{xN} + c_2 (p_{x1} + p_{x2})/\sqrt{2}$$

 p_{xN} , p_{xi} (i = 1, 2) are nitrogen and oxygen atomic p_x orbitals, respectively. The x direction is as usually chosen perpendicular to the molecular plane; the y direction is along the O-O axis.

It immediately follows that g_x approximates very closely to g_e , in fair agreement with the g(100)-value of AN1 (table 5). It can be shown that g_y (O-O direction) and g_z are both larger than g_e , also in agreement with table 5. The isotropic HF splitting should be small and due to polarization phenomena. The HF tensor should exhibit considerable anisotropy with the largest splitting along the x direction. As a result we can state that indeed the AN1 spectrum can be explained by an NO₂²⁻ model.

The spin Hamiltonian of P2 is thus also in excellent qualitative agreement with a PO_2^{2-} model, in view of the already mentioned similarity between the AN1 and P2 spin Hamiltonians. In table 5 we compare the spin-Hamiltonian data of several molecular ions isoelectronic with PO_2^{2-} . It is found that the g-tensor of P2 fits in very well. For the HF data we are faced with several possibilities for the signs of the A-values. The only signs that were determined with certainty are those for SO_2^- and SeO_2^- in KCl, and the AN1 spectrum as the corresponding isotropic spectra could be observed at higher temperatures [1-3, 28]. For the other ions we included two reasonable choices of the signs (all A-values positive or the two smallest A-values negative). For SeO_{2}^{-} we changed the sign choices mentioned in [29], yielding close agreement with the well established values for KCl:SeO₂⁻. In the original paper, two A-values were taken as negative for AsO_2^{2-} [30] while for PO_2^{2-} [7, 10] both choices occur. Using the A_0 - and B_0 -values in table 4, we calculated the s and p contributions (ρ_s and ρ_p) to the unpaired spin density for the ions in table 5. These values were plotted in figure 5 in a $\rho_{\rm p}, \rho_{\rm s}$ diagram together with a fairly representative set of experimental data for the distinct types of molecule and molecular ion that reasonably can be considered for P2 [1-10, 19, 23, 24, 28-30]. It follows that in spite of the large spread in ρ_p -values the group of defects isoelectronic with PO₂²⁻ or PO are well separated from PO₄²⁻, PO₃, PO₃²⁻ and PO₂ (solely on the base of the **A**-tensor). Taking also into account the **g**-tensor considerations about NO and NO₂²⁻ made above and in the literature [19, 22, 27] we believe that the P2 spectrum can be best explained in terms of a PO₂²⁻ model.

In the alkali halides the O_3^- , SO_2^- and SeO_2^- ions are located in an anion vacancy site. As the non-paramagnetic PO_2^- ion was already identified in unirradiated KCI [29] with an orientation in the lattice that corresponds perfectly to the **g**-tensor axes for the P2 defect, the identification as PO_2^{2-} is even corroborated. The latter paramagnetic defect can be formed by simply trapping an electron during the x-irradiation.

5. Conclusion

Irradiation of phosphate-doped KCl single crystals yields two related O⁻ centres at anion sites. We have insufficient experimental data to decide between a PO₃ or PO₄²⁻ model for a phosphorus-containing radical that is weakly detectable at RT. The second phosphorus-containing and the most interesting defect is probably a PO₂²⁻ radical in an anion vacancy, in agreement with the models established earlier for the isoelectronic O₃⁻, SO₂⁻ and SeO₂⁻ ions in alkali halides. A PO model is considered to be less probable and we agree with several workers that earlier spectra assigned to KCl:NO and KBr:NO can be more adequately interpreted in terms of an NO₂²⁻ model. In any case the hyperfine tensors of the paramagnetic defects isoelectronic with PO or PO₂²⁻ are rather sensitive to the host lattice.

Acknowledgments

The authors wish to thank Professor Dams and Dr Piens (INW) for the neutron irradiation of some of our samples. This work is part of a project sponsored by the Executieve van de Vlaamse Gemeenschap, Departement Onderwijs, which is gratefully acknowledged. F Callens is a Research Associate of the NFSR (Belgium).

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